Inference at $*$
of proof for Lemma exists_functionality_wrt_iff:

$\vdash \forall S, T$:Type, P, Q:$(S \to \mathbb{P})$.
$(S = T) \Rightarrow (\forall x{:}S.\ P(x) \iff Q(x)) \Rightarrow ((\exists x{:}S.\ P(x)) \iff (\exists y{:}T.\ Q(y)))$
by ((GenUnivCD)
CollapseTHENA ((Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n)) (first_tok :t) inil_term)))·

1:

1. S : Type
2. T : Type
3. P : $S \to \mathbb{P}$
4. Q : $S \to \mathbb{P}$
5. $S = T$
6. $\forall x{:}S.\ P(x) \iff Q(x)$
7. $\exists x{:}S.\ P(x)$

$\vdash \exists y{:}T.\ Q(y)$

2:

1. S : Type
2. T : Type
3. P : $S \to \mathbb{P}$
4. Q : $S \to \mathbb{P}$
5. $S = T$
6. $\forall x{:}S.\ P(x) \iff Q(x)$
7. $\exists y{:}T.\ Q(y)$

$\vdash \exists x{:}S.\ P(x)$

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